Lect 4

Channel capacity

Introduction

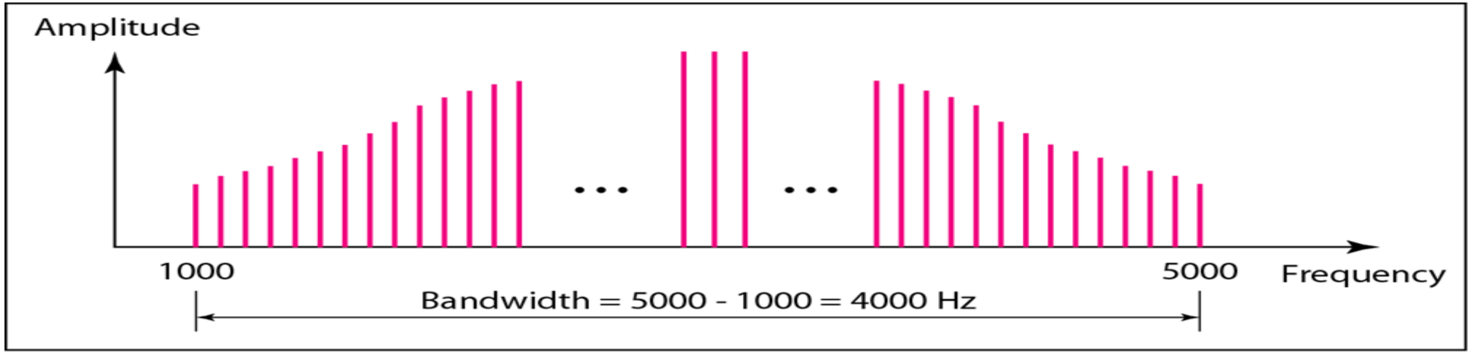
- Bandwidth can be defined as the portion of the electromagnetic spectrum occupied by the signal.
- It may also be defined as the frequency range over which a signal is transmitted.
- Different types of signals have different band

- width. Ex. Voice signal, music signal, etc.
- Bandwidth of analog and digital signals are calculated in separate ways; analog signal bandwidth is measured in terms of its frequency (hz) but digital signal bandwidth is measured in terms of bit rate (bits per second, bps)

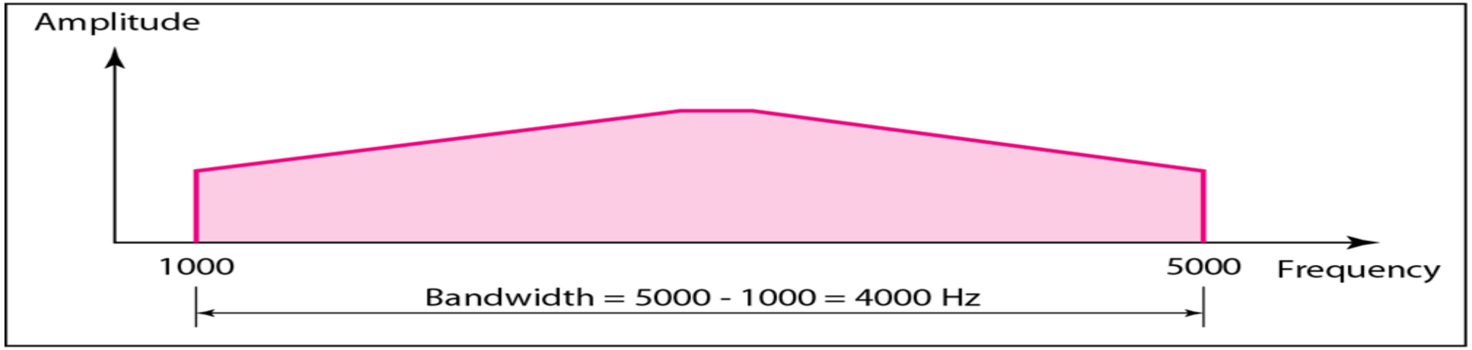
- Bandwidth of signal is different from bandwidth of the medium/channel.

Bandwidth of an analog signal

- Bandwidth of an analog signal is expressed in terms of its frequencies.
- It is defined as the range of frequencies that the composite analog signal carries.
- It is calculated by the difference between the maximum frequency and the minimum frequency.



a. Bandwidth of a periodic signal



b. Bandwidth of a nonperiodic signal

Fig: 1 bandwidth of Analog signal

Bandwidth of a digital signal

- It is defined as the maximum bit rate of the signal to be transmitted. It is measured in bits per second.
- A channel is the medium through which the signal carrying information will be passed.
- In terms of analog signal, bandwidth of the channel is the range of frequencies that the channel can carry.
- In terms of digital signal, bandwidth of the channel is the maximum bit rate supported by the channel. i.e. the maximum amount of data that the channel can carry per second.

- The bandwidth of the medium should always be greater than the bandwidth of the signal to be transmitted else the transmitted signal will be either attenuated or distorted or both leading in loss of information.

TYPES OF CHANNELS:

- Each composite signal has a lowest possible(minimum) frequency and a highest possible (maximum) frequency.
- From the point of view of transmission, there are two types of channels: -
- **Low pass Channel**
  - This channel has the lowest frequency as _0‘ and highest frequency as some non-zero frequency _f1‘.
  - This channel can pass all the frequencies in the range 0 to f1.
- **Band pass channel**
  - This channel has the lowest frequency as some non-zero frequency _f1‘ and highest frequency as some non-zero frequency _f2‘.
  - This channel can pass all the frequencies in the range f1 to f2.

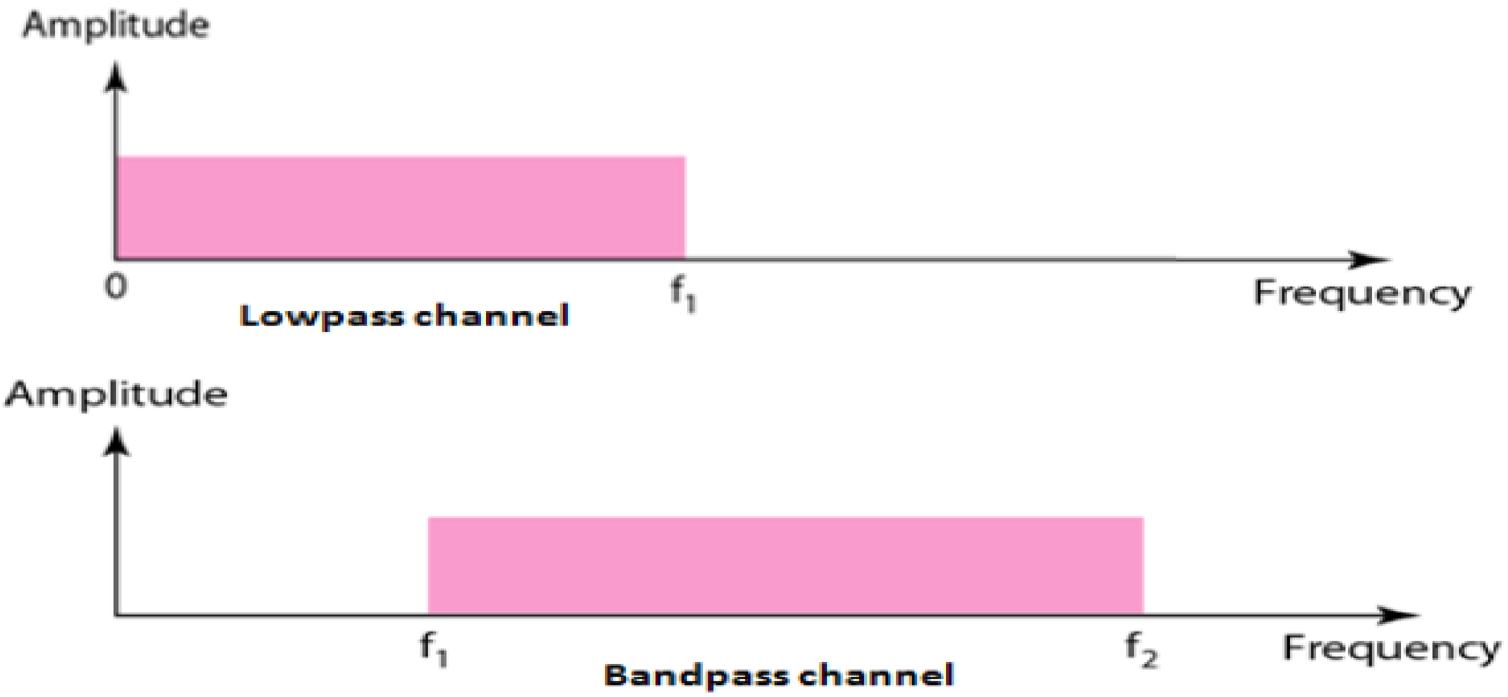


Fig5: Lowpass Channel & Bandpass Channel

Transmission of Digital signal

- Digital signal can be transmitted in the following two ways:

➤ **Baseband Transmission**

- The signal is transmitted without making any change to it (i.e. Without modulation).
- In baseband transmission, the bandwidth of the signal to be transmitted has to be less than the bandwidth of the channel.

- Ex. Consider a Baseband channel with lower frequency 0Hz and higher frequency 100Hz, hence its bandwidth is 100 (Bandwidth is calculated by getting the difference between the highest and lowest frequency).

- We can easily transmit a signal with frequency below 100Hz, such a channel whose bandwidth is more than the bandwidth of the signal is called **Wideband channel.**
- Logically a signal with frequency say 120Hz will be blocked resulting in loss of information, such a channel whose bandwidth is less than the bandwidth of the signal is called **Narrowband channel.**

➤ Broad band Transmission

- Given a bandpass channel, a digital signal cannot be transmitted directly through it
- In broadband transmission we use modulation, i.e we change the signal to analog signal before transmitting it.

- The digital signal is first converted to an analog signal, since we have a band pass channel we cannot directly send this signal through the available channel.
- Ex. Consider the band pass channel with lower frequency 50Hz and higher frequency 80Hz, and the signal to be transmitted has frequency 10Hz.

- To pass the analog signal through the bandpass channel, the signal is modulated using a carrier frequency.
- Ex. The analog signal (10Hz) is modulated by a carrier frequency of 50Hz resulting in an signal of frequency 60Hz which can pass through our bandpass channel.

- The signal is demodulated and again converted into an digital signal at the other end as shown in the figure below.

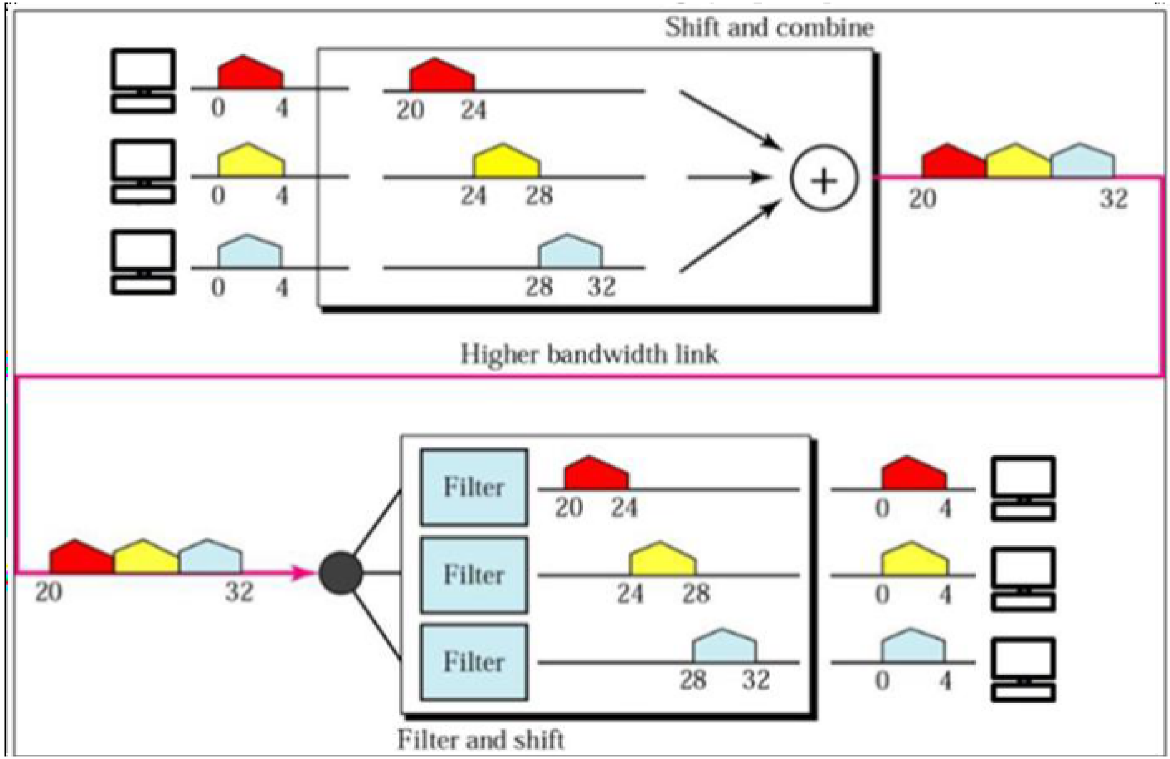


Fig6: Broadband Transmission Involving Modulation & Demodulation

Level

- Fig: A digital signal with Two levels. „1" represented by a positive voltage and „0" represented by a negative voltage

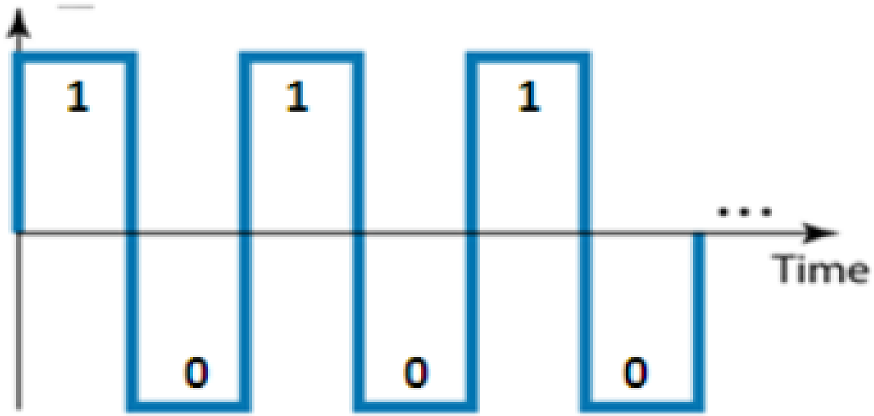


Fig: 2 digital signal with Two levels. „1" represented by a positive voltage and 0" represented by a negative voltage

A Signal can have more than two levels

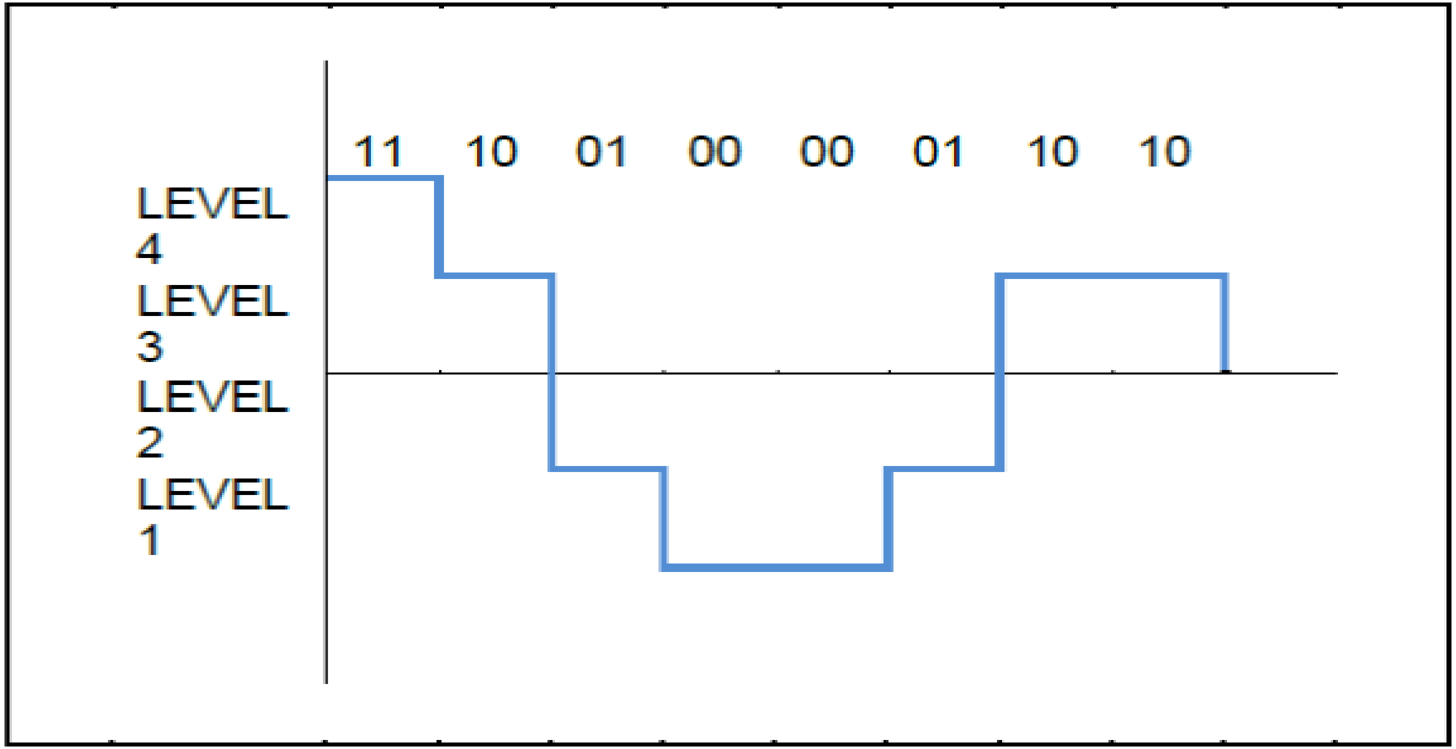


Fig3: A digital signal with four levels

- In general, if a signal has L levels then, each level need $Log_2 L$ bits • Example: Consider a digital Signal with four levels, how many bits are required per level?
- Answer:
- Number of bits per level = $Log_2 L = Log_2 4 = 2$
- Hence, 2 bits are required per level for a signal with four levels.

BIT LENGTH or Bit Interval (Tb)

- It is the time required to send one bit.
- It is measured in seconds.

BIT RATE

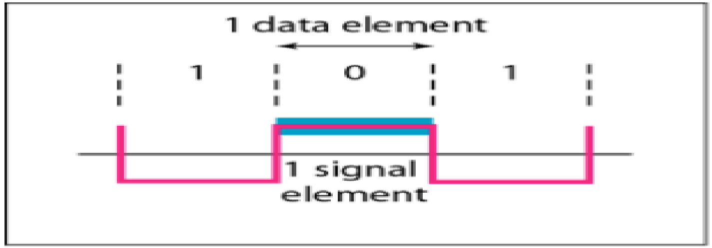
  - It is the number of bits transmitted in one second.
  - It is expressed as bits per second (bps).
- Relation between bit rate and bit interval can be as follows

Bit rate = 1 / Bit interval

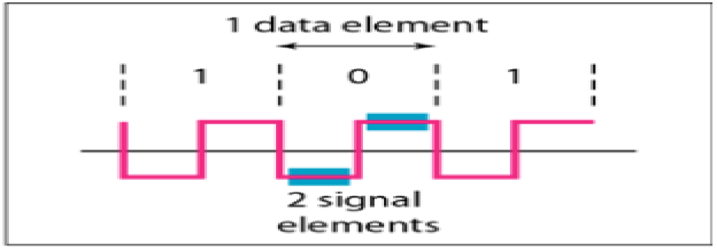
Data Element vs Signal Element

- In data communications, our goal is to send data elements.
- A data element is the smallest entity that represent a piece of information; this is the bit.
- In digital communications, a signal element carries data element.
- A signal elements is the shortest unit (timewise) of a digital signal.

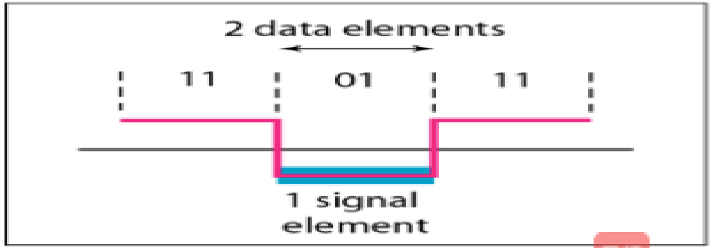
- **Data elements are being carried; signal element are the carriers.**
- We define ratio (r) which is the number of data element carried by each signal element.



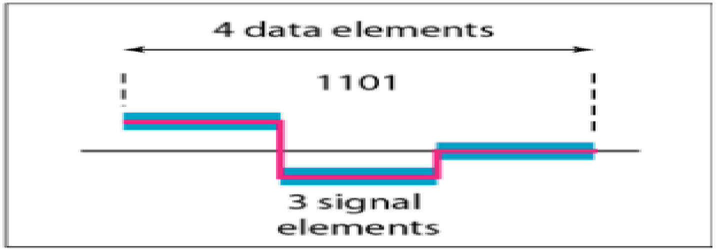
a. One data element per one signal element $(r = 1)$



b. One data element per two signal elements $\left(r = \frac{1}{2}\right)$



c. Two data elements per one signal element $(r = 2)$



d. Four data elements per three signal elements $\left(r = \frac{4}{3}\right)$

Baud Rate

- It is the rate of Signal Speed.

i.e the rate at which the signal changes.

- A digital signal with two levels ‗0’ & ‗1’ will have the same baud rate and bit rate.

The diagram below shows three signal of period (T) 1 second.

a) Signal with a bit rate of 8 bits/ sec and baud rate of 8 baud/sec.

b) Signal with a bit rate of 16 bits/ sec and baud rate of 8 baud/sec.

c) Signal with a bit rate of 16 bits/ sec and baud rate of 4 baud/sec.

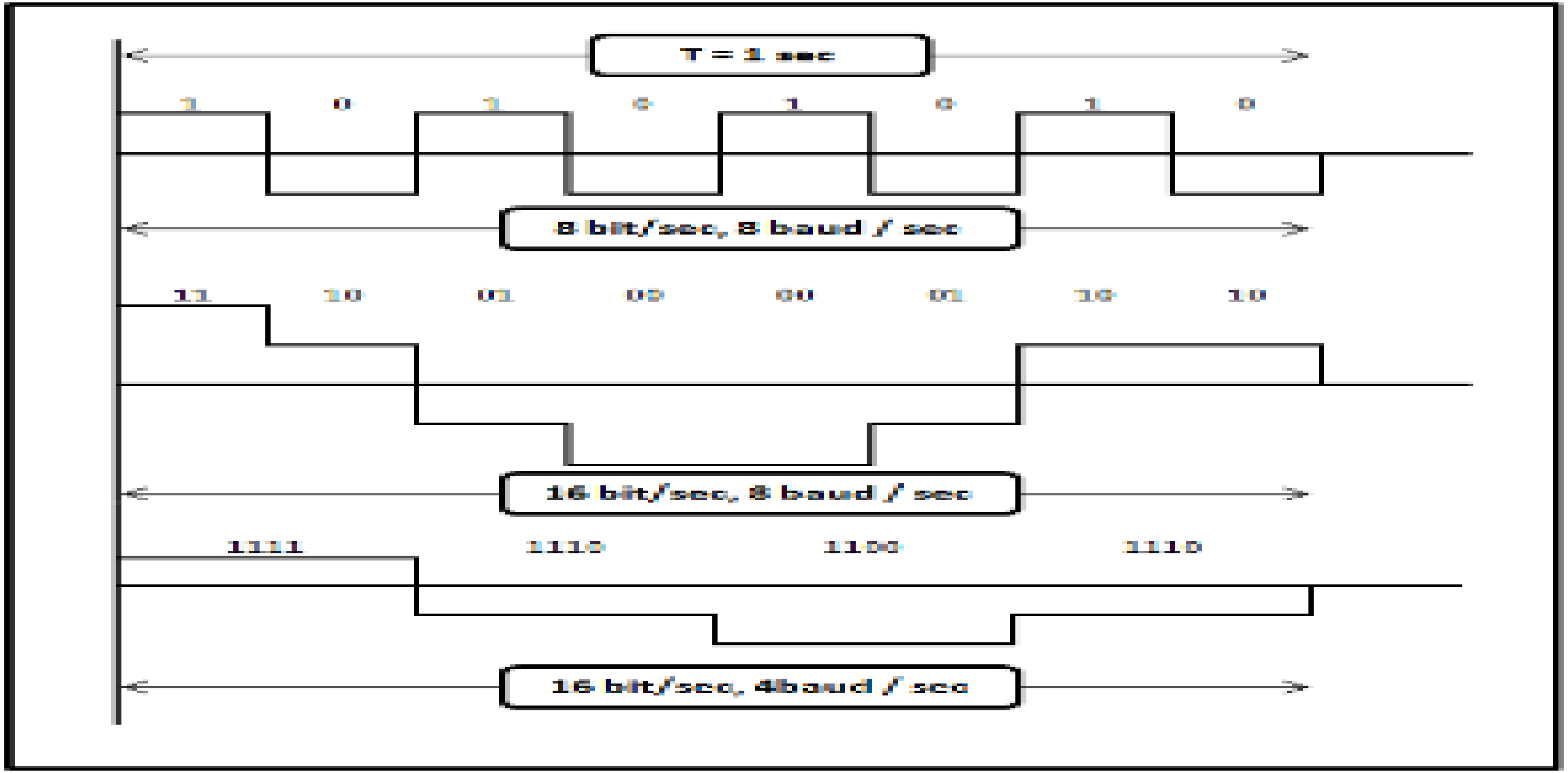


Fig4: Three signals with different bit rates and baud rates

Data Rate vs Bit Rate

- Data rate defines the number of data elements (bit) sent in 1s. The unit is bits per second (bps)
- The signal rate is the number of signal elements sent in 1 s. The unit is baud.
- The data rate is sometimes called the bit rate; the signal rate is sometimes call pulse rate, the modulation rate, or the baud rate.
- One goal in data communications is to increase the data rate while decreasing the signal rate.
- Increasing the data rate increase the speed of transmission; decreasing the signal rate decrease the bandwidth requirement.

Relationship between data rate and signal rate as:

- Where N is the data rate (bps); c is the case factor, which is varies for each case; S is the number of signal elements; and r is the previously define factor.

$$S = C \; x \; N \; x \; \frac{1}{r}$$

example

- A signal is carrying data in which one data element is encoded as one signal element (r = 1). If the bit rate is100 kbps, what is the average value of the baud rate if c is between 0 and 1?
- Solution

We assume that the average value of c is 1/2 .

The baud rate is then

s=100000x0.5x1=50000 (50kbaud)

THE MAXIMUM DATA RATE OF A CHANNEL

- Data rate depends on three factors:

1. The bandwidth available .
2. The level of the signals we use.
3. The quality of the channel (the level of noise).

Note

Increasing the levels of a signal increases the probability of an error occurring, in other words it reduces the reliability of the system. Why??

Capacity of a System

- The bit rate of a system increases with an increase in the number of signal levels we use to denote a symbol.
- A symbol can consist of a single bit or “n” bits.
- The number of signal levels = 2^n.
- As the number of levels goes up, the spacing between level decreases -> increasing the probability of an error occurring in the presence of transmission impairments.

Channel Quality:-

The quality of the channel indicates two types:

a) **A Noiseless or Perfect Channel**

- An ideal channel with no noise.
- The Nyquist Bit rate.
  - It gives the bit rate for a Noiseless Channel.

b) A Noisy Channel

- A realistic channel that has some noise.
- **The Shannon Capacity.**
  - It gives the bit rate for a Noisy Channel

Nyquist Theorem

- Nyquist gives the upper bound for the bit rate of a transmission system by calculating the bit rate directly from the number of bits in a symbol (or signal levels) and the bandwidth of the system.
- Nyquist theorem states that for a noiseless channel:

$$C = 2\,B \log_2 L$$

C= capacity in bps

B = bandwidth in Hz

L(number of level)=2^n

Example

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$\boxed{\text{BitRate} = 2 \times 3000 \times \log_2 2 = 6000 \text{ bps}}$$

Example

Consider the same noiseless channel transmitting a signal with four signal levels (for each level, we send 2 bits). The maximum bit rate can be calculated as

$$\boxed{\text{BitRate} = 2 \times 3000 \times \log_2 4 = 12{,}000 \text{ bps}}$$

Example

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. How many signal levels do we need?

Solution

We can use the Nyquist formula as shown:

$$265{,}000 = 2 \times 20{,}000 \times \log_2 L$$

$$\log_2 L = 6.625 \qquad L = 2^{6.625} = 98.7 \text{ levels}$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

Shannon’s Theorem

- Shannon’s theorem gives the capacity of a system in the presence of noise.

$$C = B \log_2(1 + SNR)$$

Example

Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity C is calculated as

$$\boxed{C = B \log_2 (1 + \mathrm{SNR}) = B \log_2 (1 + 0) = B \log_2 1 = B \times 0 = 0}$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

Example

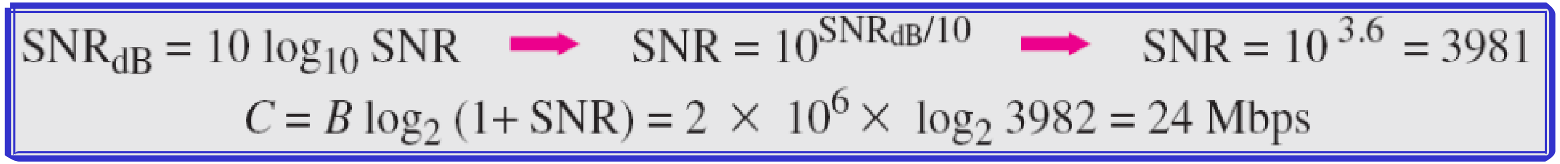
We can calculate the theoretical highest bit rate of a regular telephone line. A telephone line normally has a bandwidth of 3000. The signal-to-noise ratio is usually 3162. For this channel the capacity is calculated as

$$C = B \log_2 (1 + \text{SNR}) = 3000 \log_2 (1 + 3162) = 3000 \log_2 3163$$
$$= 3000 \times 11.62 = 34{,}860 \text{ bps}$$

This means that the highest bit rate for a telephone line is 34.860 kbps. If we want to send data faster than this, we can either increase the bandwidth of the line or improve the signal-to-noise ratio.

Example

The signal-to-noise ratio is often given in decibels. Assume that SNR_{dB} = 36 and the channel bandwidth is 2 MHz. The theoretical channel capacity can be calculated as



$$SNR_{dB} = 10 \log_{10} SNR \rightarrow SNR = 10^{SNR_{dB}/10} \rightarrow SNR = 10^{3.6} = 3981$$

$$C = B \log_2 (1 + SNR) = 2 \times 10^6 \times \log_2 3982 = 24 \text{ Mbps}$$

$$SNR_{dB} = 10 \log_{10} SNR$$

Example

For practical purposes, when the SNR is very high, we can assume that SNR + 1 is almost the same as SNR. In these cases, the theoretical channel capacity can be simplified to

$$\boxed{C = B \times \frac{\mathrm{SNR_{dB}}}{3}}$$

For example, we can calculate the theoretical capacity of the previous example as

$$\boxed{C = 2\ \mathrm{MHz} \times \frac{36}{3} = 24\ \mathrm{Mbps}}$$

Example

We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution

First, we use the Shannon formula to find the upper limit.

$$\boxed{C = B \log_2 (1 + \text{SNR}) = 10^6 \log_2 (1 + 63) = 10^6 \log_2 64 = 6 \text{ Mbps}}$$

Example (continued)

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$\boxed{4\text{ Mbps} = 2 \times 1\text{ MHz} \times \log_2 L \quad \longrightarrow \quad L = 4}$$

Note

The Shannon capacity gives us the upper limit; the Nyquist formula tells us how many signal levels we need.