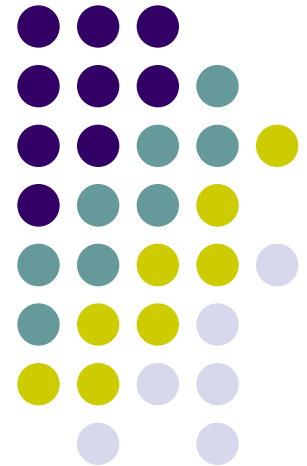


# ITMC403 Parallel and Distributed Computing

## Dependence analysis

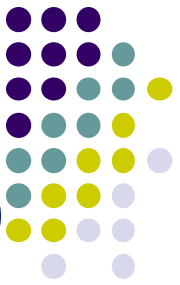




# Fundamental Assumption

- **When can two statements execute in parallel?**
  - **On one processor:**
    - statement 1;
    - statement 2;
  - **On two processors:**

● <u>processor 1:</u>	<u>processor 2:</u>
● statement 1;	statement 2;
- Processors execute **independently**: no control over order of execution between processors



# Fundamental Assumption (Cont.)

- **When can two statements execute in parallel?**

- **On two processors:**

- **Possibility 1**

- processor 1:  
statement 1;  
-----

- processor 2:  
-----  
statement 2;

- **Possibility 2**

- processor 1:  
-----  
statement 2;

- processor 2:  
statement 1;  
-----

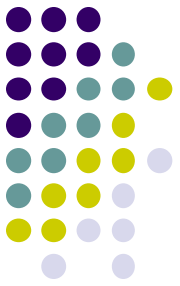
*Their order of execution must not matter!* In other words,

- statement1; statement2;

- **must be equivalent to**

- statement2; statement1;

# Examples



- **EXAMPLE 1**

- $a = 1;$  →
- $b = 2;$

Statements can be executed in parallel.

- **EXAMPLE 2**

- $a = 1;$  →
- $b = a;$  →

Statements cannot be executed in parallel  
Program modifications may make it possible.

- **EXAMPLE 3**

- $a = f(x);$  →
- $b = a;$  →

May not be wise to change the program  
(sequential execution would take longer).

- **EXAMPLE 4**

- $b = a;$  →
- $a = 1;$

Statements cannot be executed in parallel.

- **EXAMPLE 5**

- $a = 1$  →
- $a = 2$

Statements cannot be executed in parallel.



# Types of Dependences

**True (flow) dependence –RAW**      read after write

- Statements  **$S_1$** ,  **$S_2$** 
  - **$S_2$**  has a **true dependence** on  **$S_1$** 
    - **iff**
    - **$S_2$**  reads a value written by  **$S_1$**
- Denoted by  **$S_1 d S_2$**

## Example:

The first statement writes into a location that is read by the second.

**$S_1$**        $X = \dots$

**$S_2$**        $\dots = X$

We write  **$S_1 d S_2$** .



# Types of Dependences (cont.)

## Anti-dependence –WAR

write after read

- Statements **S1**, **S2**
  - **S2** has a **anti-dependence** on **S1**
    - **iff**
    - **S2** writes a value read by **S1**
- Denoted by **S1**  $d^{-1}$  **S2**

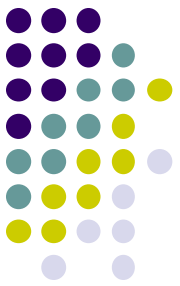
### Example:

The first statement reads from a location into which the second statement writes.

**S<sub>1</sub>**     ... = X

**S<sub>2</sub>**     X = ...

An anti-dependence is denoted by **S<sub>1</sub>**  $d^{-1}$  **S<sub>2</sub>**



# Types of Dependences (cont.)

## Output dependence –WAW

write after write

- Statements **S1**, **S2**
  - **S2** has a **output dependence** on **S1**
    - iff
  - **S2** writes a value written by **S1**
- Denoted by  $S1 \ d^0 \ S2$

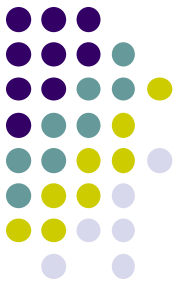
### Example:

both statements write into the same location.

**S<sub>1</sub>**      $X = \dots$

**S<sub>2</sub>**      $X = \dots$

We write  $S_1 \ d^0 \ S_2$



When can 2 statements execute in parallel?

**S1** and **S2** can execute in parallel

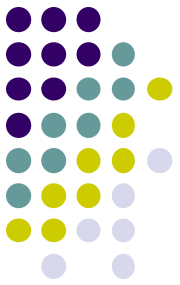
iff

there are **no dependences** between **S1** and **S2**

- **true dependences**
- **anti-dependences**
- **output dependences**

Some dependences can be removed.





# Data Dependence in Loops

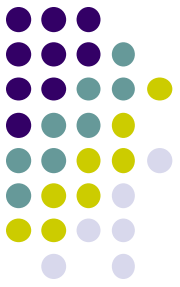
Parallelism often occurs in loops.

```
for(i=0; i<100; i++)  
    a[i] = i;
```

- No dependences.
- Iterations can be executed in parallel.

# Data Dependence in Loops

## (cont.)



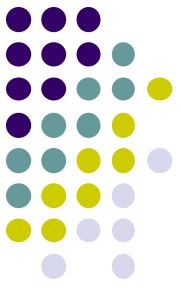
Parallelism often occurs in loops.

```
for(i=0; i<100; i++) {  
    a[i] = i;  
    b[i] = 2*i;  
}
```

Iterations and statements can be executed in parallel.

# Data Dependence in Loops

## (cont.)



Parallelism often occurs in loops.

```
for( i=1; i<100; i++ )  
    a[i] = f(a[i-1]);
```

- Dependence between  $a[i]$  and  $a[i-1]$ .
- Loop iterations are not parallelizable.

# Data Dependence in Loops

## (cont.)



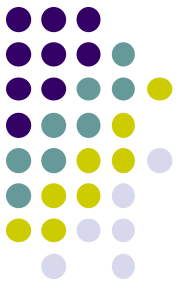
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# Data Dependence in Loops

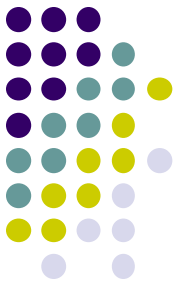
## (cont.)



- **Loop-Carried Dependence**

- A **loop-carried dependence** is a dependence that is present only if the statements occur in two different instances of a loop
- Otherwise, we call it a **loop-independent dependence**
- Loop-carried dependences limit loop iteration parallelization

# Data Dependence in Loops (cont.)

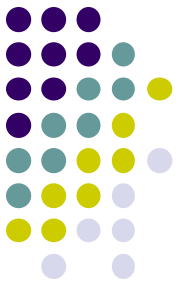


- **Loop-Carried Dependence**

```
for(i=1; i<100; i++ )  
    for(j=1; j<100; j++ )  
        a[i][j] = f(a[i][j-1]);
```

- Loop-independent dependence on i.
- Loop-carried dependence on j.
- Outer loop can be parallelized, inner loop cannot.

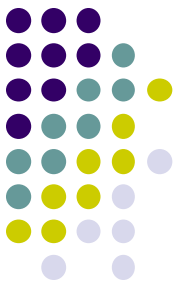
# Data Dependence in Loops (cont.)



- **Loop-Carried Dependence**

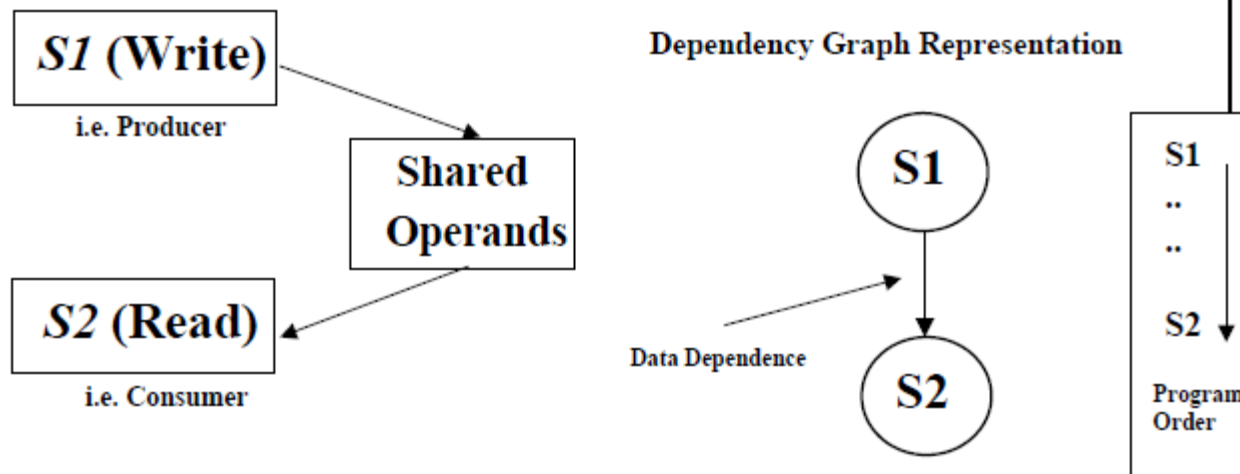
```
for( j=1; j<100; j++ )  
    for( i=1; i<100; i++ )  
        a[i][j] = f(a[i][j-1]);
```

- Inner loop can be parallelized, outer loop cannot.
- Less desirable situation (finer-grain parallelism).
- Loop interchange is sometimes possible.



## (True) Data (or Flow) Dependence

- Assume task S2 follows task S1 in sequential program order
- Task S1 produces one or more results used by task S2,
  - Then task S2 is said to be data dependent on task S1
- Changing the relative execution order of tasks S1, S2 in the parallel program violates this data dependence and results in incorrect execution.



**Task S2 is data dependent on task S1**



