Social Networking

الشبكات الاجتماعية

ITMC 413

SN As Graph

إعداد

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Basic Concepts of Social Network Analysis

Social network is formally referred to as a set of social actors, or nodes, members that are connected by one or more types of relations

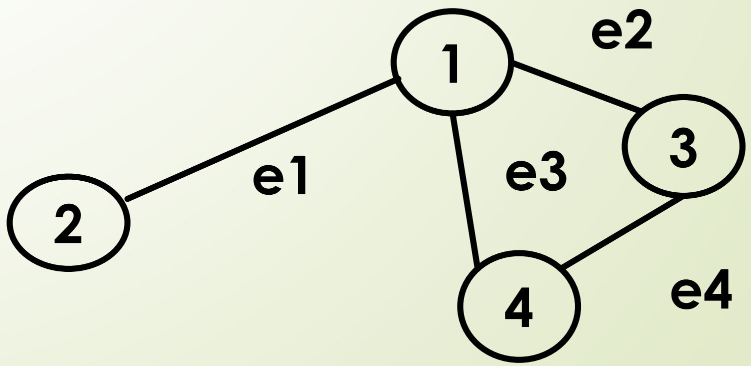
- Ties : Ties or links connect two and more nodes in a graph.
- Density : Density is one of the most basic measures in network analysis.
- Path, Length, and Distance
- Centrality: Degree Centrality, Between-ness Centrality, Closeness Centrality.

SN As Graph

- The two most common way to represent social network are the use of a graph and a matrix.
- An graph (or network) G= (V,E) consists of a set of vertices (or nodes) V together with an edge set E .The elements of E are called edges or links .we often say G is a graph on V
- The graph is a drawing of a social network in which nodes

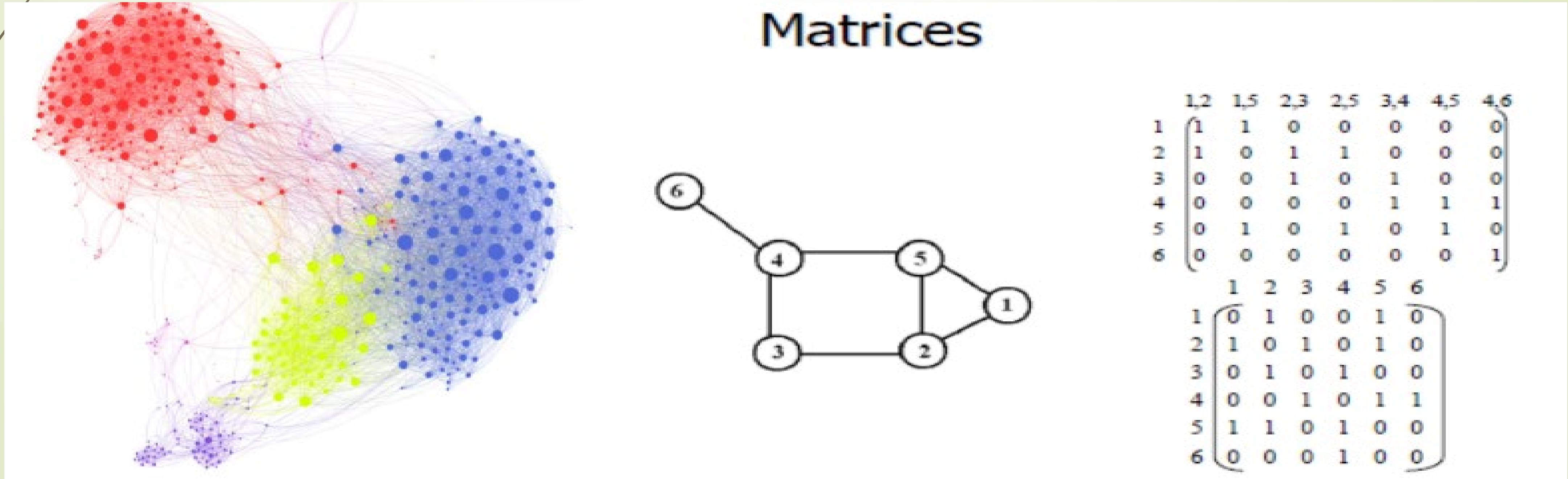
(individuals) are depicted as dots and ties (relations) are depicted as lines.

- In the following graph G consists of a set of vertices (or nodes) V together with an edges set E .Here is an example with
- V={ 1,2,3,4}, E={e1,e2,e3,e4 }



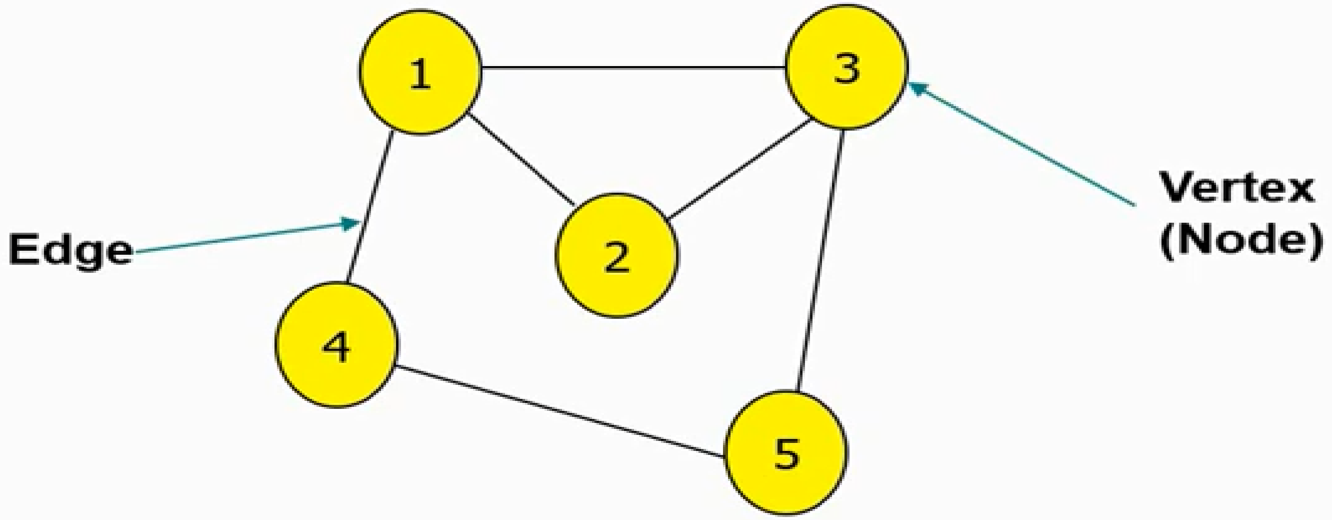
SN as Matrix

- Graphs are very useful ways of presenting information about social network .However, when there are many nodes and/or many kinds of relations, they can become so visually complicated that it is very difficult to see patterns. It is also possible to represent information about social networks in the form of matrices.



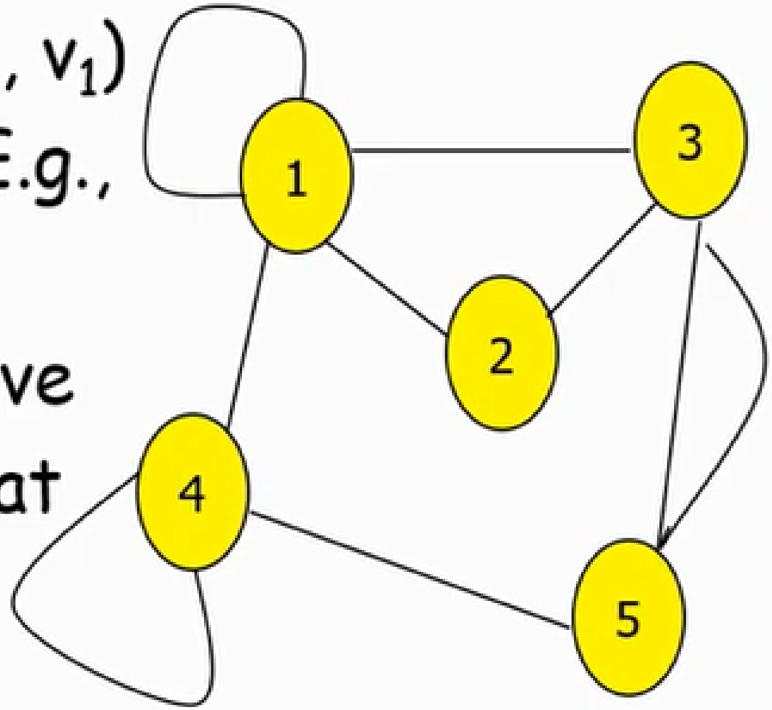
- A graph with infinite number of vertices or edges is called infinite graph.
- A graph with finite number of vertices as well as finite number of edges is called finite graph.

- For example,
- V = {1, 2, 3, 4, 5}
- E = { (1,2), (1,3), (1,4), (2,3), (3,5), (4,5) }



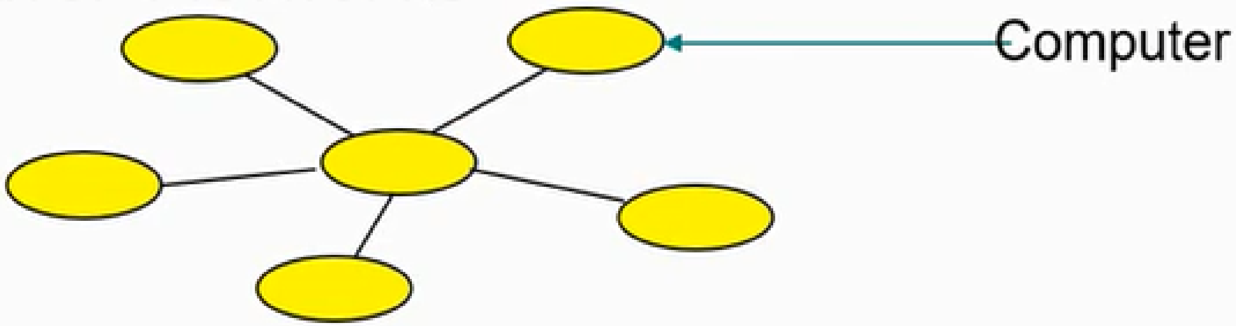
Loop or self edges

- Loop or self edges: An edge (v_1, v_1) is called a self edge or a loop. E.g., (1,1) and (4,4) are self edges
- If the same pair of vertices have more than one edges, (3, 5), that graph is called a Multigraph
- Graphs that may include loops, and possibly multiple edges connecting same pair of nodes, are called pseudographs.
- In simple graphs (or graphs) loops and multiple edges are *not* permitted

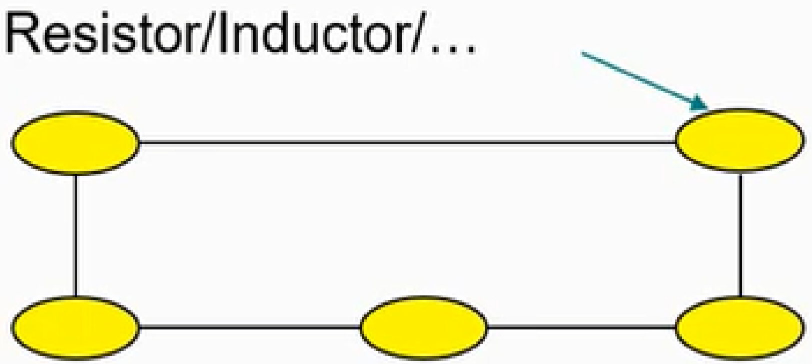


Application of Graphs

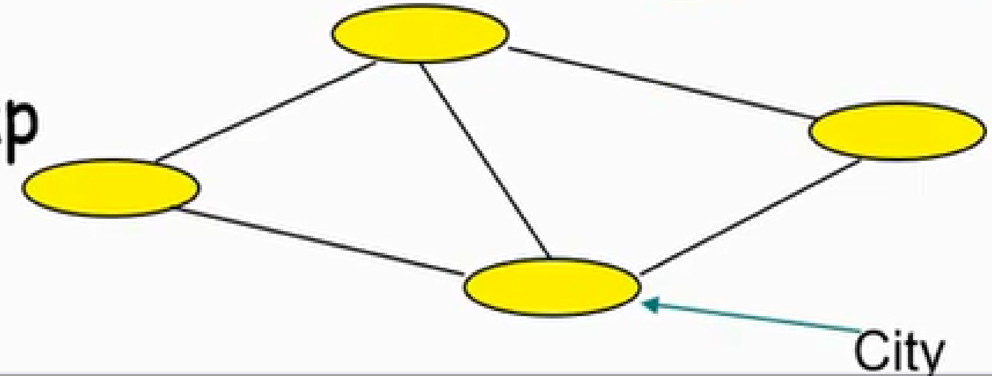
- Computer Networks



- Electrical Circuits

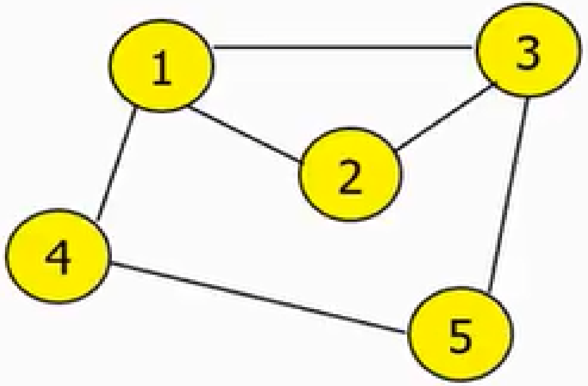


- Road Map

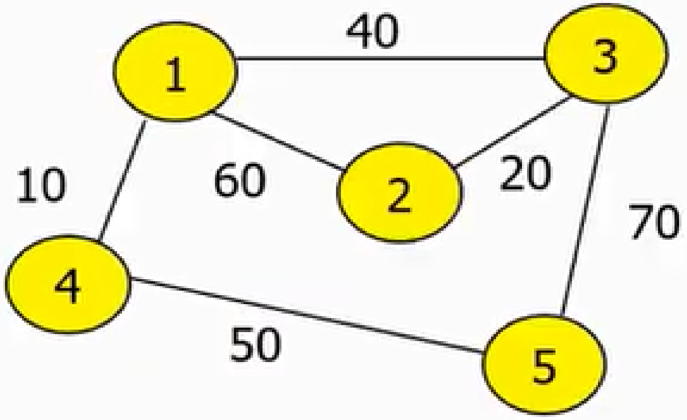


Undirected Graph

- An Undirected Graph is a graph where the edges have no directions.
- The edges in an undirected graph are called Undirected Edges

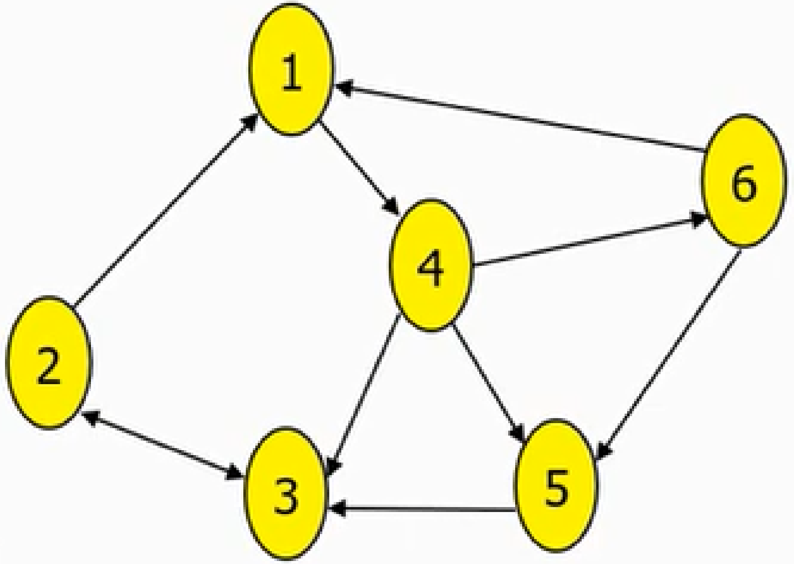


- For example, G = (V, E), where
  V = {1, 2, 3, 4, 5} and E = {(1,2), (1,3), (1,4), (2,3), (3,5), (4,5)}
- A Weighted Graph is a graph where all the edges are assigned weights.



Directed Graph

- A Directed Graph or Digraph is a graph where each edge has a direction.
- The edges in a digraph are called Arcs or Directed Edges



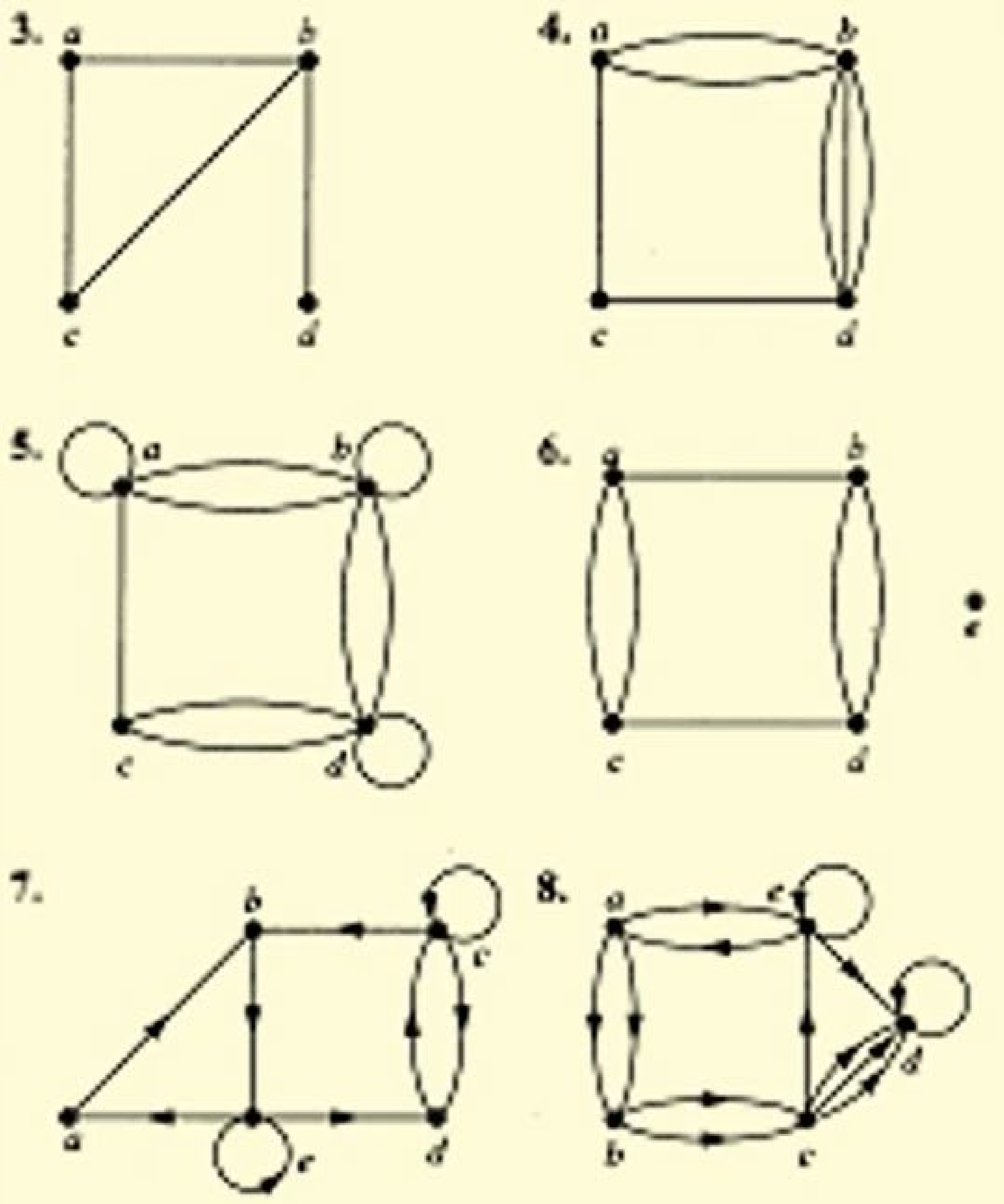
- For example, G = (V, E), where V = {1, 2, 3, 4, 5, 6} and E = {(1,4), (2,1), (2,3), (3,2), (4,3), (4,5), (4,6), (5,3), (6,1), (6,5)}
- The edge (1, 4) = 1→4 where 1 is the tail and 4 is the head

Graph Terminology

TABLE 1 Graph Terminology.			
Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Determine the type of the graph:

(Simple graph, Multigraph, Pseudograph, Simple directed graph, Directed Multigraph,Mixed graph)

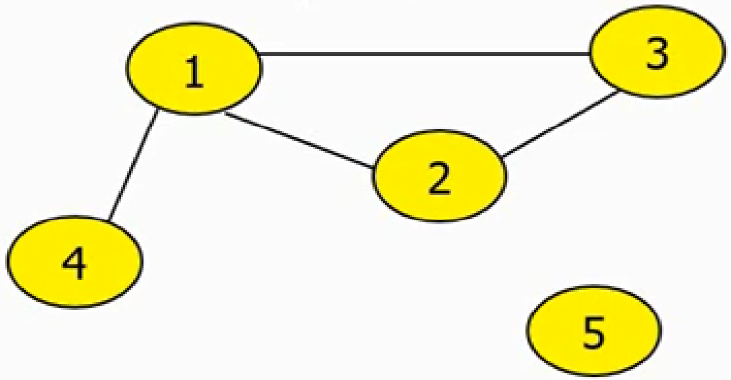


Different Measures of Social Network

Node Degree (Measures Based on Connection
In Social Networks, it can be useful to target individuals with high degree to spread information throughout the network.
The degree of node is defined as the number of edges connected to that vertex over the number of possible edges

Degree

- Degree of a Vertex : In an undirected graph, the number of edges incident to the vertex is called the degree. Degree of vertex v is denoted by d(v).
- For example, d(1)=3, d(2)=2, d(3)=2, d(4)=1 and d(5)=0.

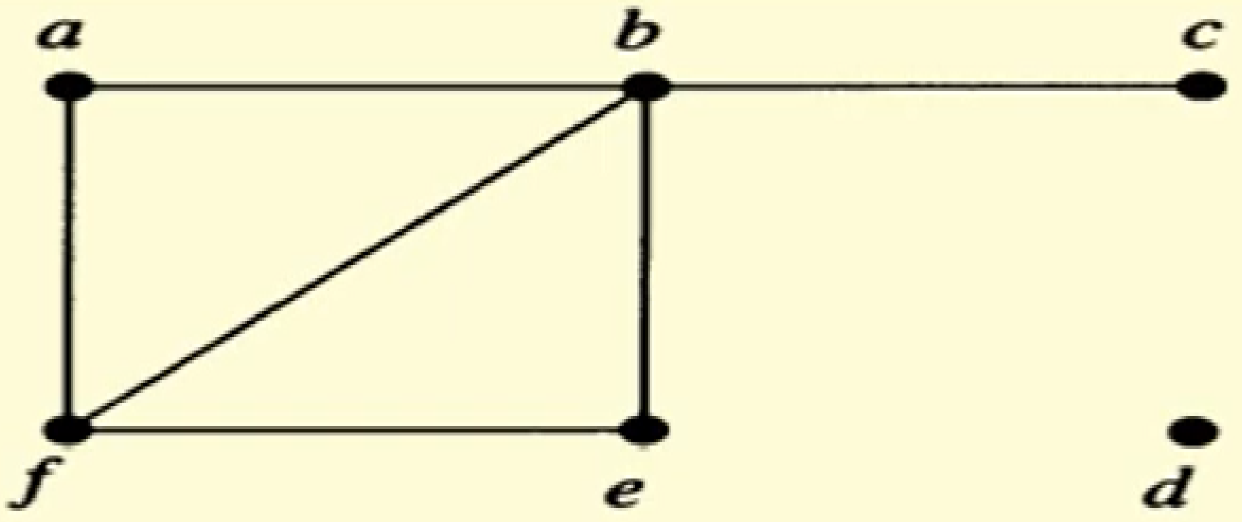


- A vertex of degree zero is called isolated vertex.
- A vertex of degree one is called pendant vertex.
- For example, vertex 4 is pendant and vertex 5 is isolated.
- A graph without any edge is called null graph.
- A graph in which all vertices are of equal degree is called regular graph.

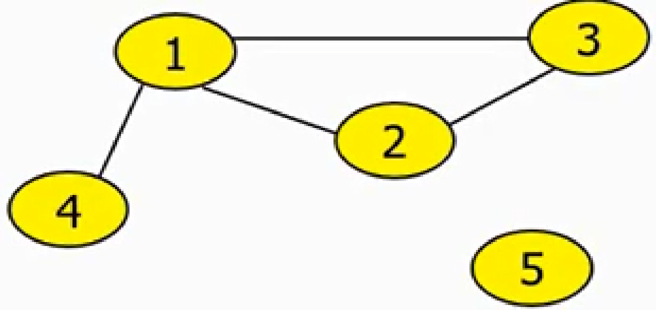
Exercise

- Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

1.



Degree



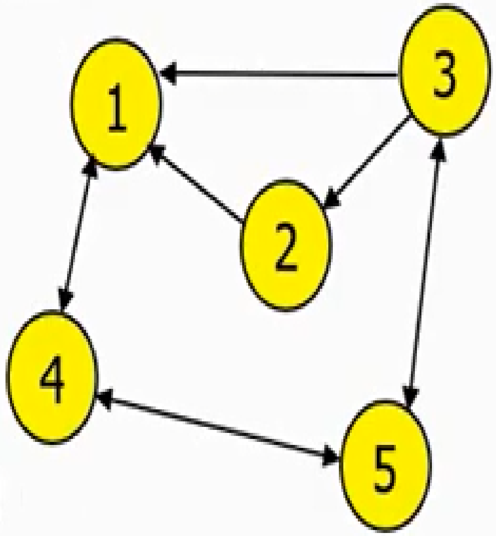
- The Handshaking Theorem: - Let $G=(V, E)$ be an undirected graph. Then $2|E| = \sum d(v), \forall v \in V$
- Example:- Taking the example in the last slide,

  $d(1) + d(2) + d(3) + d(4) + d(5)$

  $= 3 + 2 + 2 + 1 + 0 = 8$ = twice number of edges.
- Example:- How many edges are there in a graph with 10 vertices each of degree six?
- Solution:- Because the sum of the degrees of the vertices is $6 \times 10 = 60$, it follows that $2|E| = 60$. Therefore, $|E| = 30$.
- Exercise:- Can a simple graph exist with 15 vertices each of degree five?

Degree

- In diagram, there are two types of degree of a vertex : in-degree and out-degree



- In-degree: In-degree of a vertex v, denoted by $d^{-}(v)$, is the no. of edges entering the vertex in a digraph
- Out-Degree: Out-degree of a vertex v, denoted by $d^{+}(v)$, is the no. of edges leaving the vertex
- For example, $d^{-}(1)=3$ and $d^{+}(1)=1$; $d^{-}(2)=1$ and $d^{+}(2)=1$.

Exercise

Draw these graphs.

a) K_7 b) $K_{1,8}$ c) $K_{4,4}$

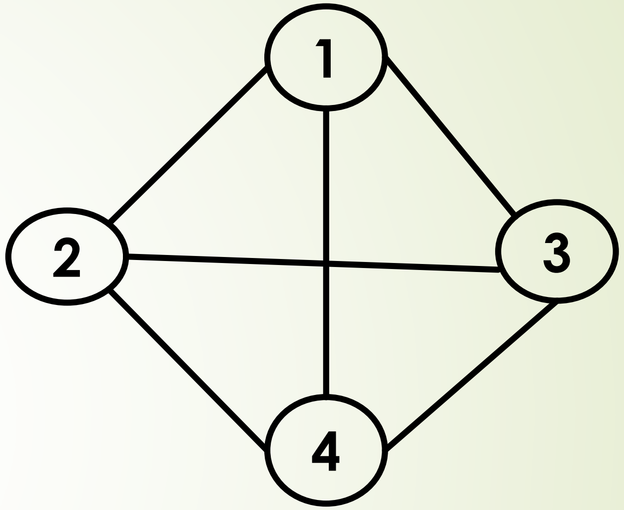
d) C_7 e) W_7 f) Q_4

K->Complete graph	C->Cycle graph N>=3	W->Wheel graph N>=3

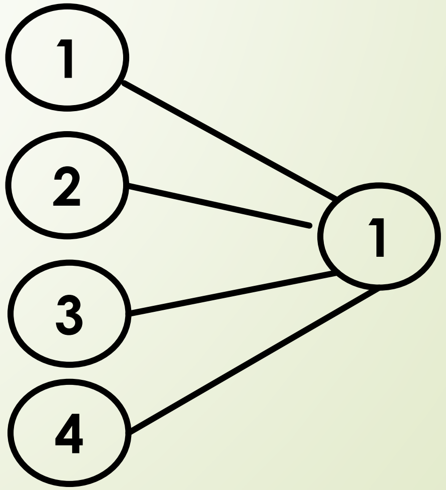
Q->n-cube graph
2^n

Draw these graphs

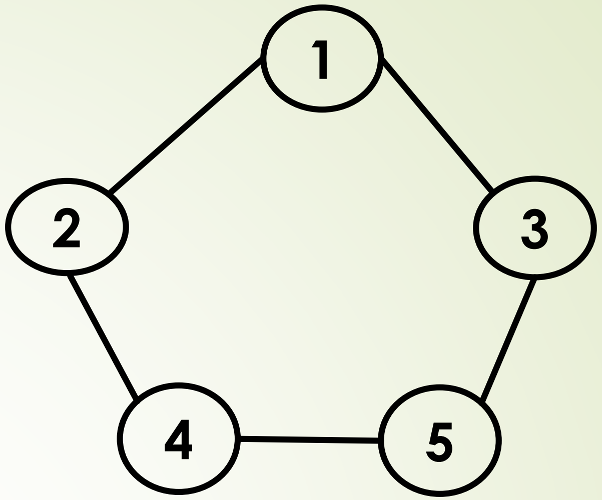
- K_4



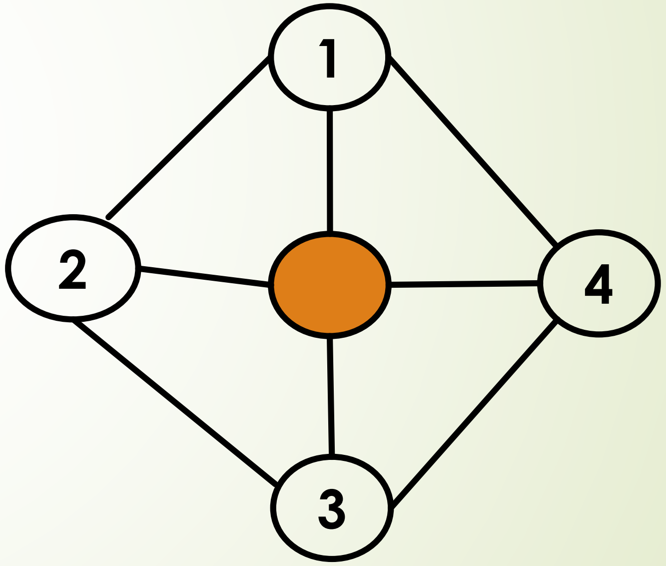
- $K_{4,1}$



- C_5



- W_4



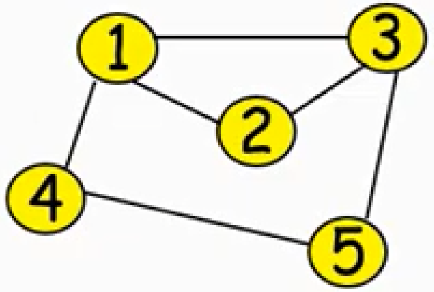
- $Q_4 = 2^4$
  $=16$

Graph Representation

- **Adjacency Matrix:-** The Adjacency Matrix $A=(a_{i,j})$ of a graph $G=(V,E)$ with n nodes is an nXn zero-one matrix. Element of A can be defined as follows:

  $$a_{ij} = 1, \text{ if } (v_i, v_j) \in E,$$
  $$= 0, \text{ otherwise}$$

- **Example:-** Find the adjacency matrices of the following graphs.



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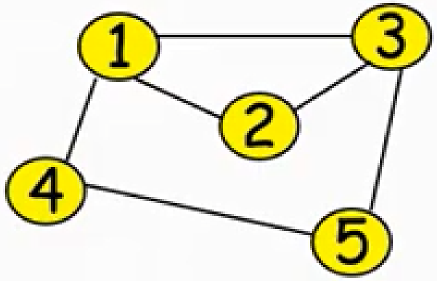
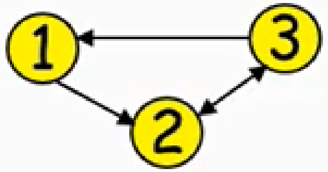


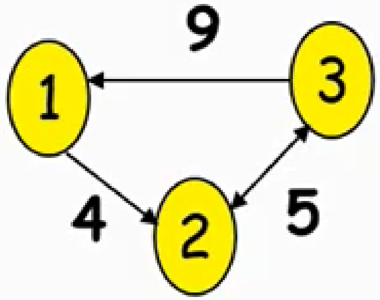
	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	0	0
3	1	1	0	0	1
4	1	0	0	0	1
5	0	0	1	1	0



	1	2	3
1	0	1	0
2	0	0	1
3	1	1	0

Graph Representation

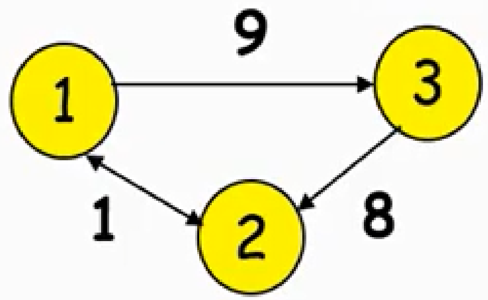
- **Adjacency Matrix** of a **Weighted Graph**:- The weight of the edge can be shown in the matrix when the vertices are adjacent. A nil value (0 or ∞) depending on the problem is used when they are not adjacent
- **Example**:- Adjacency matrix to find the minimum distance between nodes



	1	2	3
1	∞	4	∞
2	∞	∞	5
3	9	5	∞

- **Example**:- Draw graph with the adjacency matrix

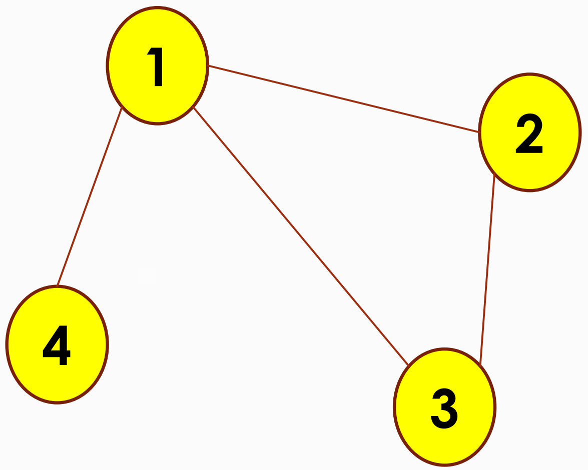
	1	2	3
1	∞	1	9
2	1	∞	∞
3	∞	8	∞



Exercise

- Use an adjacency matrix to represent the graph

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$



Graph Representation

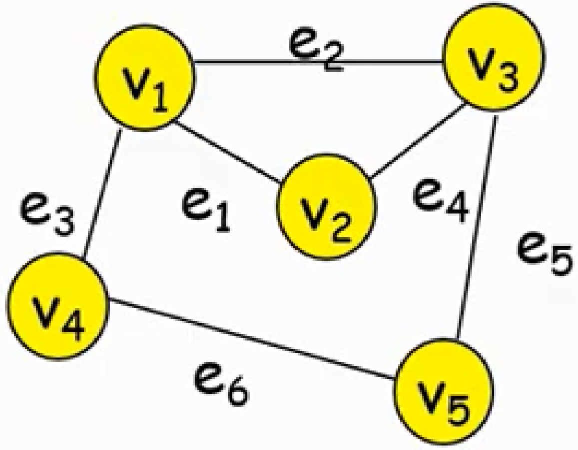
- **Incidence Matrix:-** Let $G=(V,E)$ be a graph with v_1, v_2, …, v_n are vertices and e_1, e_2, …., v_m are edges. Then the incidence matrix with respect to this ordering of V and E is the nxm matrix $M=[m_{ij}]$, where

  m_{ij} = 1, when edge e_j is incidence on v_i,

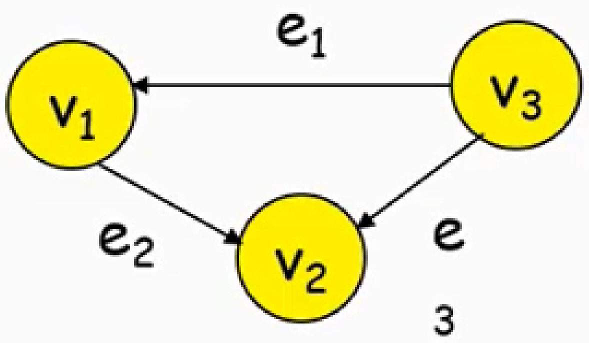
  = 0, otherwise

Graph Representation

- **Example**:- Find the incidence matrices of the following graphs.



	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	1	0	0	0
v_2	1	0	0	1	0	0
v_3	0	1	0	1	1	0
v_4	0	0	1	0	0	1
v_5	0	0	0	0	1	1



	e_1	e_2	e_3
v_1	1	0	0
v_2	0	1	1
v_3	0	0	0

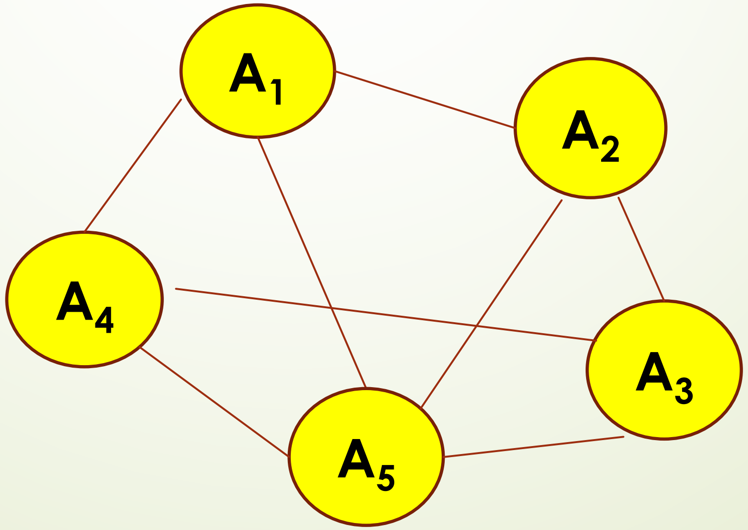
Exercise

- Represent the pseudograph using the following incidence matrix

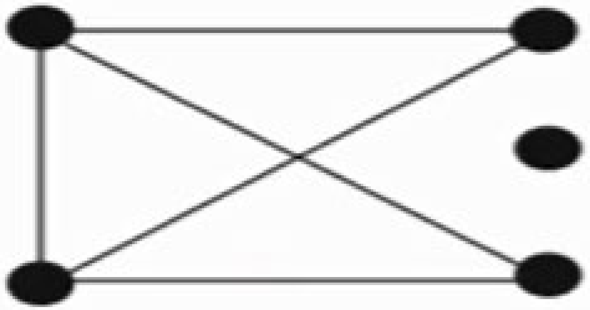
$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}.
$$

The **intersection graph** of a collection of sets $A_1, A_2, \ldots, A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

a) $A_1 = \{0, 2, 4, 6, 8\}$, $A_2 = \{0, 1, 2, 3, 4\}$, $A_3 = \{1, 3, 5, 7, 9\}$, $A_4 = \{5, 6, 7, 8, 9\}$, $A_5 = \{0, 1, 8, 9\}$

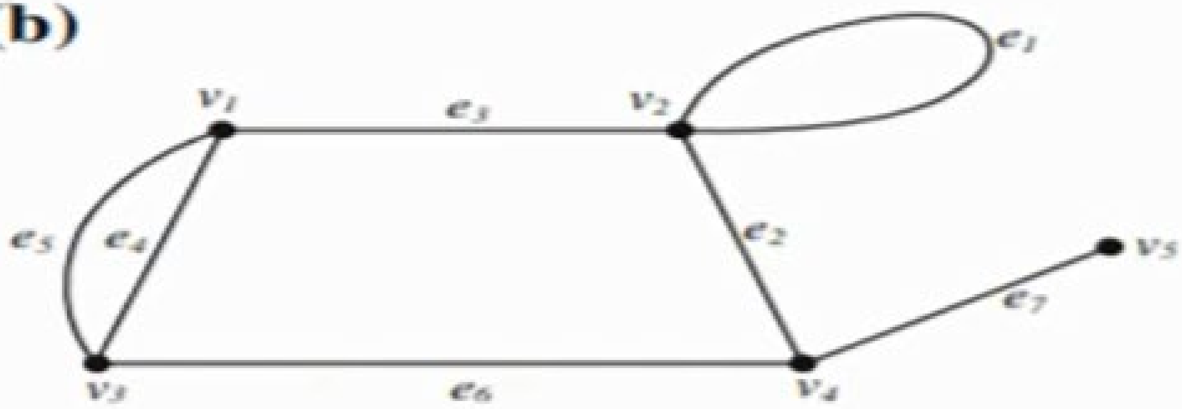


(a)



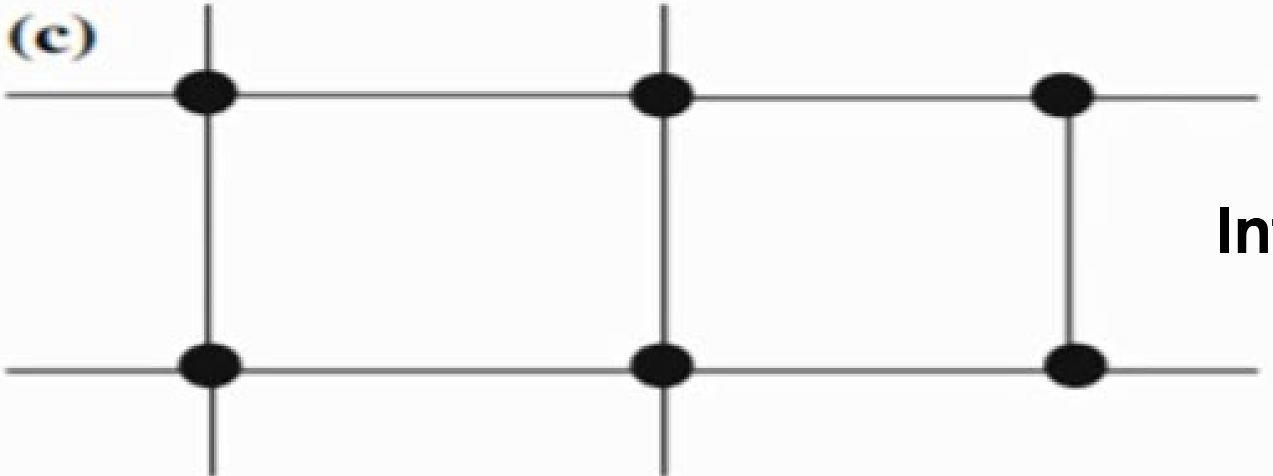
Simple graph

(b)



Pseudo graph

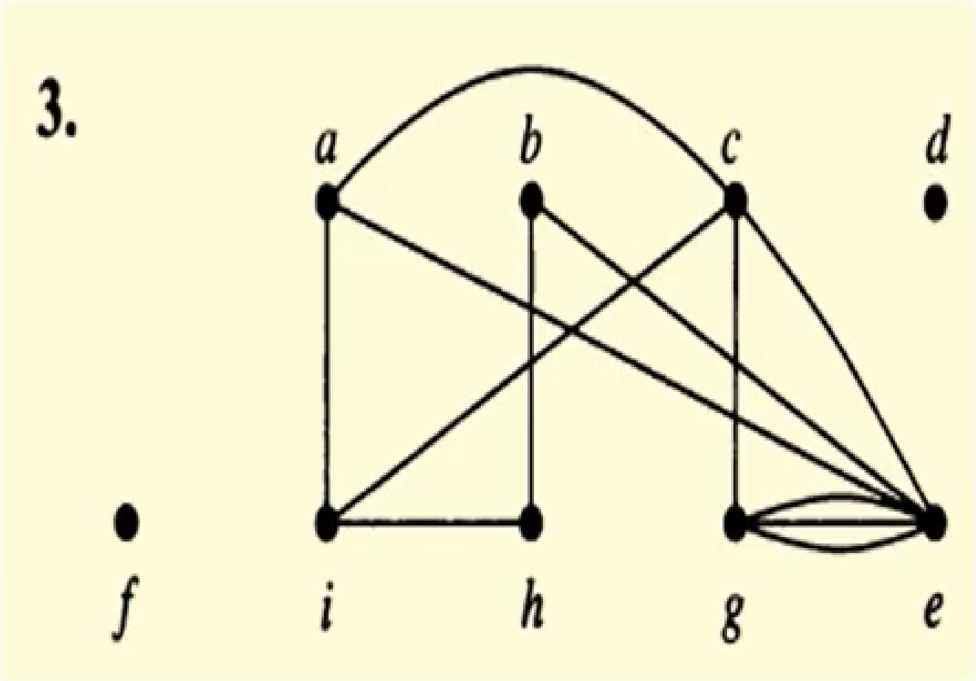
(c)



Infinite graph

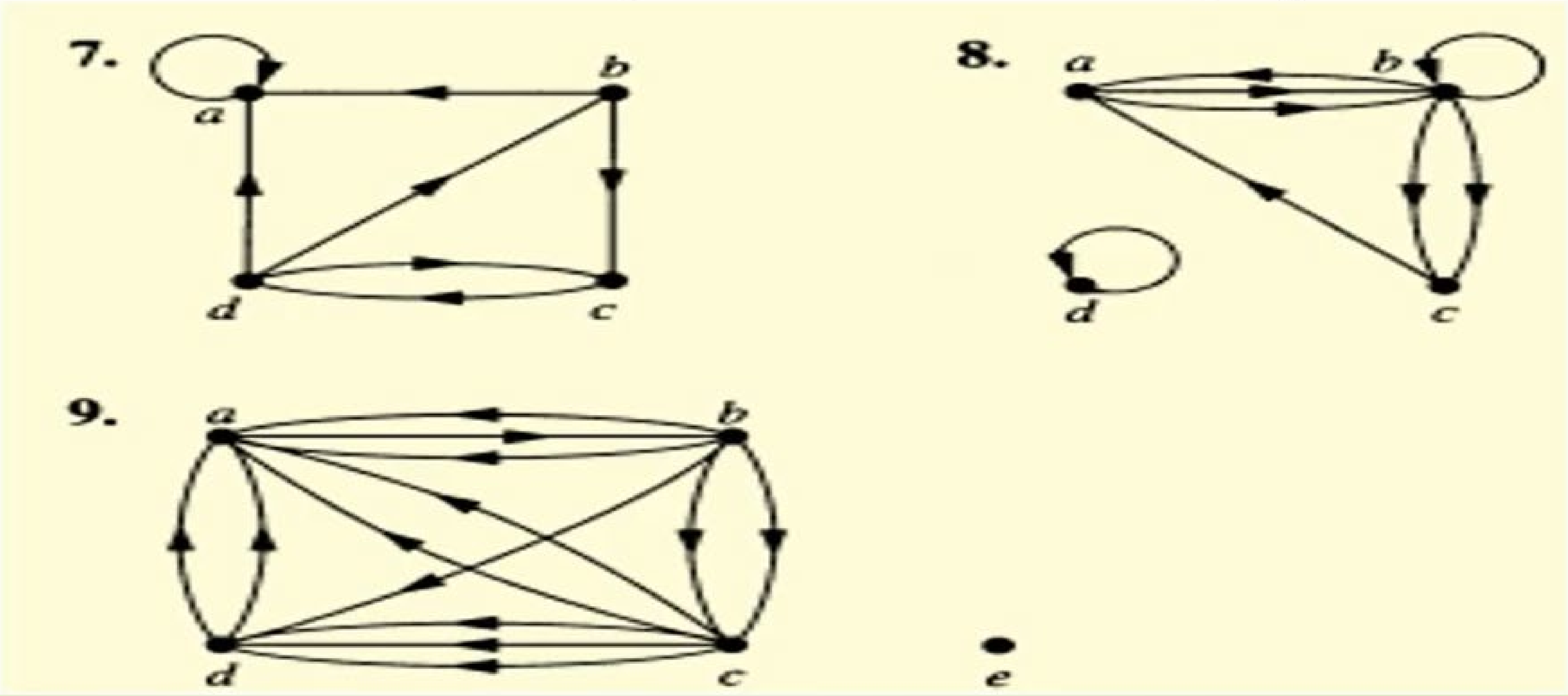
Exercise

- Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



Exercise

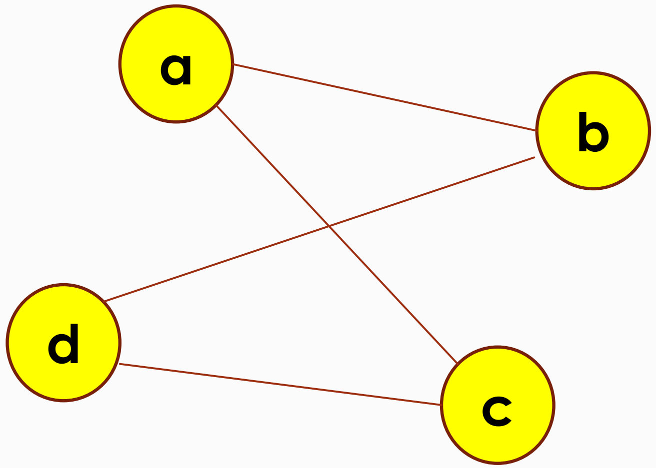
- In Exercises 7-9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



Exercise

- Draw a graph with the adjacency matrix with respect to the ordering of vertices a, b, c, d.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$



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